Electromagnetic lenses

The electromagnetic lenses are used to decrease the crossover dimensions of the electron beam. For a W source the crossover is reduced from a dimension of about 50 μm to a spot size on the sample of about 10 nm (demagnification of 5000x).

For a system with a field emission source, the crossover dimension is just reduced and for this reason a maximum 100x of demagnification is enough to obtain a spot size of 1-2 nm.

Generally speaking the electrons can be focused by electrostatic fields as well as by electromagnetic fields.

Nevertheless, SEM microscope is provided by electromagnetic lenses because they are less affected by aberration effects (which are higher than that ones presented in the optic lenses).
The electrostatic lens

**Fig. 4.1.** Examples of electrode systems producing axially symmetrical electrostatic fields.
## The magnetic lens

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Electromagnetic Lens

The lens is formed by a solenoid of N coils where a $I$ current is flowing. This current produces a magnetic field with cylindrical symmetry which is concentrated in a small volume by the use of a ferromagnetic material (soft iron). Moreover ferromagnetic polepieces further concentrate the magnetic field in a small region.

The polepieces are drilled in the center to allow the passage of the electrons which travel along the axis of the lens.

These polepieces are machined to high precision to ensure that the magnetic field has the high degree of axial symmetry required for good focusing.
The polepieces concentrate the magnetic flux lines with a high degree of axial symmetry and the electrons are deflected by the magnetic field according to the Lorentz law. Since the field is proportional to NI, you can control the focus of the lens by controlling the current passing through it.
An electron beam, entering the lens in a direction parallel to the axis, is focused at a point (focus) whose distance from the center of the two polepieces is called the focal distance and represents the "strength of a lens".
Optical and Electromagnetic lenses

$f$ depends on the curvature ray of the lens and it is fix. It is possible to change the lenses distance to modify the focus distance.

In the electromagnetic lenses the focal distance is proportional to the acceleration voltage and depends on the intensity and shape of the field along the axis of the lens. Since the focusing field extends only a few millimeters along the axis, so that to a first approximation the lens may be considered thin and it is possible to apply the Gauss formula of the conjugate points of the thin optical lens.
Beam focusing

In analogy with the conjugate points equation of the thin optical lens:

\[ \frac{1}{f} = \frac{1}{p} + \frac{1}{q} \]

**Magnification:** \( M = \frac{q}{p} \)

**Demagnification:**

\[ m = \frac{p}{q} \quad \Rightarrow d_1 = \frac{d_0}{m} \]

where \( p \) is the distance from the object to the lens center while \( q \) is the distance from the image to the lens center.

Using more lenses (2/3) it is possible to further focus the beam by obtaining a spot size of 10 nm (excluding the aberrations) from a tungsten source.
Electromagnetic Lenses in a SEM column

**Condenser lenses:** usually the SEM condenser system is formed by 1-3 condenser lenses for reducing the electron beam dimension. The first condenser focuses the beam. The second condenser is usually synchronized with the first one and, in this way, they can be simultaneously controlled.

**Objective lens:** this last lens focuses the beam on the sample making a further reduction of the beam. Since the objective is a strong focusing lens (a high current flows in the coils of this lens), a cooling system is usually provided.

The objective lens is characterized by an aperture which limits the beam convergence to limit the lens aberrations. In fact, the objective lens induces higher aberrations because it strongly reduces the electron beam.
Lenses control

The operator can change three parameters to control the electron beam characteristics:

• To change the objective aperture.
• To fix the working distance by changing the sample position on z axis.
• To change the first condenser parameters.

Effects due to the objective aperture

The aperture decreases the divergence angle from $\alpha_1$ to $\alpha_2$ and it has three important effects on the final beam:

• To find the best angle to reduce the aberrations.
• To control the field depth.
• To determine the final current beam since only a fraction of the electron beam goes through the aperture and arrives on the sample.
Lenses control

Effects due to the working distance (W)

W is the distance from the objective to the sample surface. It can be changed by moving the sample along the z axis and refocusing the beam. The W increase induces the $q_2$ increase and a reduction of

$$m_2 = \frac{p^2}{q^2}$$

Moreover, the increase of spot size gets worst the image resolution, even if the beam current remains the same. Finally, the $\alpha_2$ angle reduction increases the depth of focus.
Lenses control

Effects due to the first lens condenser

The first lens condenser has effect both on the dimension and the current of the electron beam. A weak focusing power of the condenser induces a greater current which passes through the aperture. For this reason the beam spot size on the sample are relatively high. Fixing the W and the objective aperture, the increase of the focusing power of the first condenser induces an increase of the focusing power of the other lenses reducing the spot size on the sample.

Nevertheless, this is responsible of a beam current decrease.

For this reason, it is necessary to find the better compromise between the current and the spot size of the beam in each specific case.
The beam current versus spot dimensions

Let’s consider a Gaussian beam not affected by aberrations.

From the brightness definition, it is possible to derive the equation of the spot size $d_G$ (at FWHM):

$$d_G = \sqrt{\frac{4i_p}{\beta \pi^2 \alpha_p^2}}$$

From the above equation, the current beam on the sample $i_p$ is given by:

$$i_p = \frac{\beta \pi^2 \alpha_p^2 d_G^2}{4}$$

If aberrations are not present in the system, one could increase $\alpha_p$ for $i_p$ increasing at a fix spot size of the beam. Nevertheless, $\alpha_p$ has to be remain as small as possible due to the aberrations and for this reason $i_p$ is not so high.
Aberrations

All lenses have defects / aberrations, but unlike the optical lenses, the aberrations of the electromagnetic lenses cannot be eliminated by using combinations of several lenses with different properties. The only possibility is to minimize the effects.

Aberrations can be classified as following:

• spherical aberration
• diffraction aberration
• chromatic aberration
• astigmatism.

The effects of aberrations are more important to the objective lens which reduces the electron beam to the smallest sizes.
The electron trajectories arriving at the lens at larger distances from the optic axis are focused to points that differ from the focal point of the paraxial trajectories.

Each image point becomes a disk of confusion, and the image produced on any given plane is blurred (reduced in resolution).
**Spherical aberration**

The electrons from the point P traveling along the trajectory PA are focused in the Gaussian image plane, as expected in the absence of aberrations. The electrons traveling along the trajectory PB, with the maximum permitted divergence from the opening of the lens, are focused at a point closer to the lens. This gives rise to an image that has the shape of a disk rather than a point. The smallest disk, close to the image plane, it is often called spherical aberration disk of least confusion.

The spot size of such disk is given by:

\[ d_s = \frac{1}{2} C_s \alpha^3 \]

where \( C_s \) is the spherical aberration coefficient (which depends on \( V_0 \) and \( f \)) and \( \alpha \) is the angle formed by the outsider ray.

The contribution of \( d_s \) to the final spot size could be reduced decreasing \( \alpha \) by modifying the lens aperture.

Unfortunately, very small apertures reduce excessively the current and introduce diffraction aberration.
Diffraction aberration

For the wave nature of the electrons, when the rays emanating from a point pass through a lens of semiangular aperture $\alpha$, they form an image which is no longer a point but with the intensity spread out in what is known as an Airy disk.

The distance between two minima on either side of the main intensity peak is given by:

$$d_d = \frac{1.22\lambda}{\sin \alpha}$$

The contribution to spot size is taken (according to Wells' indications, 1974) as half of the diameter of the first minimum quantified by the equation:

$$d_d = \frac{0.61\lambda}{\alpha}$$

where $\lambda$ is the wavelength associated to electrons and $\alpha$ is the angle of convergence.
Diffraction aberration

When two emitting points of the object lie very close together the intensity patterns will overlap. The resolution of the system in terms of Airy disk separation is defined as the distance between the maxima when the maximum intensity from one point is coincident with the first minimum from the other point. The separation of the two points is then:

\[ r = \frac{0.61 \lambda}{\sin \alpha} \]

This resolution limitation, called the diffraction limit, is due to the size of the electrons wavelength and of the convergence angle.
Compromise between spherical aberration and diffraction aberration

From the equation: \[(514,323),(676,350) \]
\[ \alpha \]

It is evident that the higher \( \alpha \), the lower is the contribution of \( d_d \).

For this reason, spherical aberrations (\( d_s = \frac{1}{2} C_s \alpha^3 \)) and diffraction aberrations change in opposite direction with respect to \( \alpha \).

It is necessary to find an optimal value of \( \alpha \) (\( \alpha_{opt} \)) that it is the right balance between the two effects.
Chromatic aberrations

An optical lens gives rise to images with coloured fringes due to the different focal lengths for red and blue light due to the variation of the refractive index of the lens material with wavelength.

The defect can be largely corrected by forming a compound lens employing glass of different refractive indices, but the highest resolution is obtained by using monocromatic illumination.
Chromatic aberrations

The chromaticity of the electron beam is determined by the energy spread of the electrons within the beam ($\Delta E$), since this gives a spread of electron wavelengths which are focused in different points of the optic axis with the formation of a disk in the image plane. The spot size of the chromatic disk of confusion is given by:

$$d_c = C_c \alpha \frac{\Delta E}{E_0}$$

dove $C_c$ is the chromatic aberration coefficient and $\Delta E/E_0$ is the ratio of the energy spread and the kinetic energy of the electron beam.
Example

The energy spread ($\Delta E$) of an electron beam from a tungsten source is of about 3 eV.

\[
\frac{\Delta E_0}{E_0} = 10^{-4} \quad @ \; 30 \; \text{keV}
\]
\[
\frac{\Delta E_0}{E_0} = 10^{-3} \quad @ \; 3 \; \text{keV}
\]

So the effects of chromatic aberration is 10 times higher at 3 keV than that at 30 keV.

Moreover, $C_c$ is almost proportional to the focal length and, in the objective lens, has the same value of $C_s$. It points out that $C_c$ could be reduced by stabilizing the accelerating voltage and the current of the lenses.
Astigmatism

The mechanical defects, inhomogeneities in the polepieces and the irregularities of the edges of the lenses are responsible of the astigmatism. This implies that the electrons emerging from P are focused differently on 2 mutually perpendicular planes. The result is that a object point is imaged at two focal lines, and the smallest circular image of the object point is halfway between the focal lines.

Astigmatism stretches the image of the object point in two perpendicular directions, depending on whether the image is above or below the focus. Stretching vanishes on the focus.

Shape of the beam section:
(a) Before the astigmatism correction
(b) Above the focus
(c) Beyond the focus
(d) Corrected image
Astigmatism

Fortunately, it is a defect which can be corrected without too much difficulty by using a stigmator which eliminates the ellipticity and makes the beam symmetric.

The stigmator consists of an electrostatic octopole or a magnetic quadrupole which produce electric or magnetic fields of intensity and direction appropriate to control the beam.

Since astigmatism can be completely corrected in a SEM fully functional, only spherical and diffraction aberrations should be corrected at the normal acceleration voltages (15-30 kV). Chromatic aberrations begin to seriously degrade the image for acceleration voltages below 10 kV.
Spot dimensions versus the beam current

**Minimum probe size**: the spot size of the electron beam \( (d_p) \) is calculated by summing the square of the Gaussian beam diameter \( (d_c) \) and the diameters of the disks of the all the aberrations with the following equation:

\[
d_p = \left(d_G^2 + d_s^2 + d_d^2 + d_c^2\right)^{1/2}
\]

It is possible to verify that \( d_p = d_{\text{min}} \) at \( \alpha_{\text{opt}} \).

At the typical SEM accelerating voltage (15-30 kV) the relation between the spot size and the current of the electron beam can be found at \( \alpha_{\text{opt}} \) by considering only the spherical and diffraction aberration and neglecting the chromatic aberration:

\[
d_p = d_{\text{min}} = KC_s^{1/4} \lambda^{3/4} \left(\frac{i_p}{\beta \lambda^2} + 1\right)^{3/8}
\]

where \( K \) is a constant approximately equal to 1, \( d_{\text{min}} \) unit is nm, if \( C_s \) and \( \lambda \) are in nm, and \( i_p \) in Amps (A).

For \( i_p \) near to zero \( d_{\text{min}} \) is \( KC_s^{1/4} \lambda^{3/4} \) which is considered the theoretical resolution of the instrument.
The beam current versus spot dimensions

Maximum probe current: at the typical SEM accelerating voltage (15-30 kV), neglecting the chromatic aberration and fixing the aperture at $\alphaopt$, the maximum current is given by:

$$i_{\text{max}} = \frac{3\pi^2}{16} \beta \frac{d_p^{8/3}}{C_z^{2/3}}$$

Note: from the brightness definition, it is possible to derive the equation of the beam current without considering the aberrations contribution:

$$i_p = \frac{\beta \pi^2 \alpha_p^2 d_G^2}{4}$$

The aberrations contribution increases the dependence of the current beam to the spot size from $d^2$ to about $d^3$.

Considerations:

The decrease of the spot size improves the resolution but induces a high current decreasing. Nevertheless, an increase of the accelerating voltage and then of the brightness, produces an increment on the excitation volume of the sample, worthing the spatial resolution. Another possibility to increase the beam current consists on the reduction of the working distance.